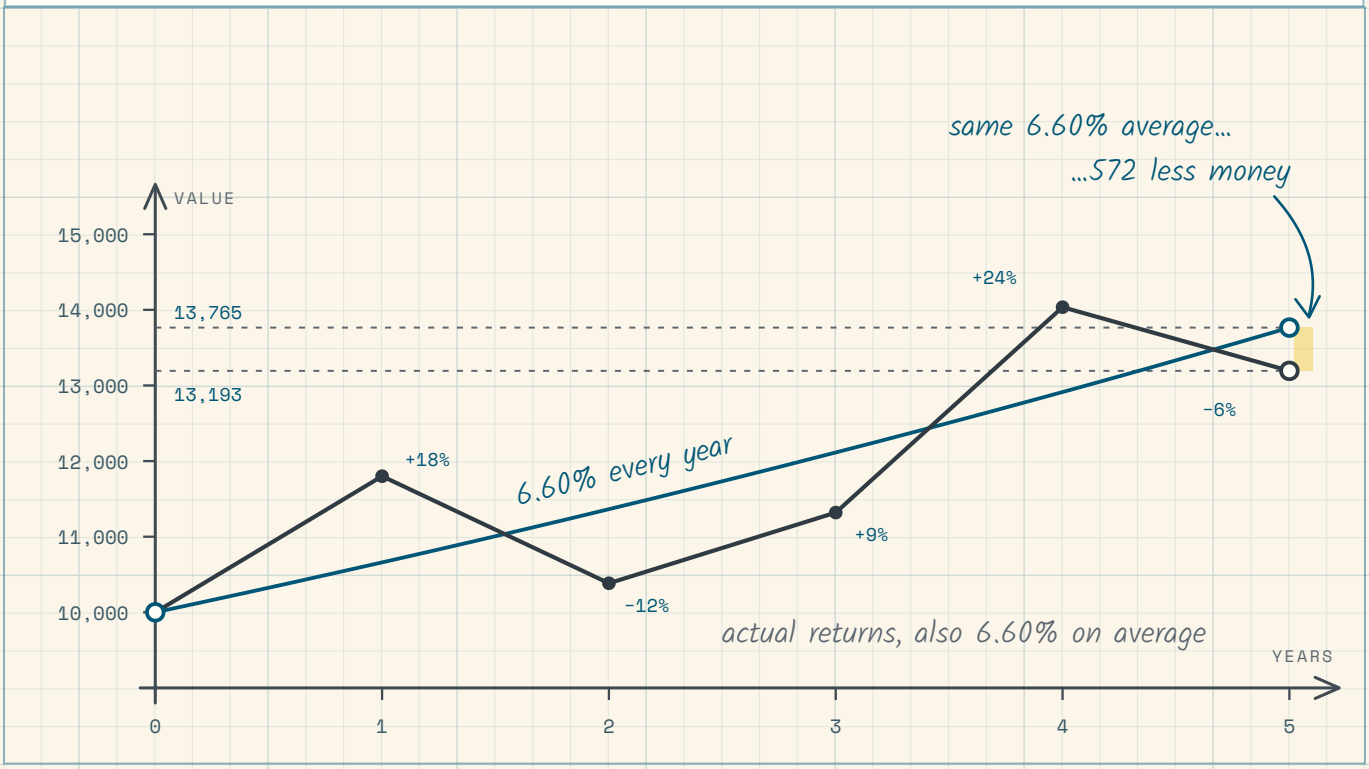
LEARNING GUIDE · CFA® LEVEL I · 2027 CURRICULUM

Returns of Financial Assets and Instruments

Quantitative Methods, Learning Module 1. The whole module on squared paper: every formula boxed, every number worked by hand, every trap circled.

19 formulas | 15 worked examples | 14 practice questions | BA II Plus & HP 12C keys

SAME AVERAGE, LESS MONEY — 10,000 INVESTED FOR 5 YEARS



Both average 6.60% a year. Only the geometric mean, 5.70%, tells you where the money ends up. — Fig. 2.2

BEFORE YOU START

How to read these notes

Think of this guide as the notes a strong candidate keeps in the margins of the curriculum. Typeset text explains each idea. Handwriting works the numbers.

price return	A term the exam expects you to define and tell apart from its neighbors.
EQ. 5	A boxed formula. Learn it: all 19 are collected on the formula sheet, p. 20.
= 4.80 / 48.00	Working, written out the way you would write it on exam day.
0.0032	A ringed number: where the trap is hiding.
12.50% ✓	The answer, double underlined, then a check you can do in ten seconds.
EXAM TRAP	A wrong answer the exam likes to offer, and why it is wrong.
	Fore-edge tabs for sections 1–6, the toolkit and practice. In the PDF they are links.

what the exam asks →

LOS

What the curriculum asks of you

- ☐ **describe, compare, and interpret returns**
  §1 Where returns come from · §2 Averages · §3 Time · §5 Adjusted · §6 Leverage

- ☐ **describe, compare, and interpret required rates of return, risk-free rates, risk premia, and inflation**
  §4 Required return, real risk-free rate, premia, excess returns

tick each box when you can explain it out loud. Outcome 2 is in the official outline, though the reader omits its checkbox

The outcome verbs are *describe, compare, interpret*, yet the module's own examples and end-of-module questions are mostly calculations. Be ready to compute everything in this guide.

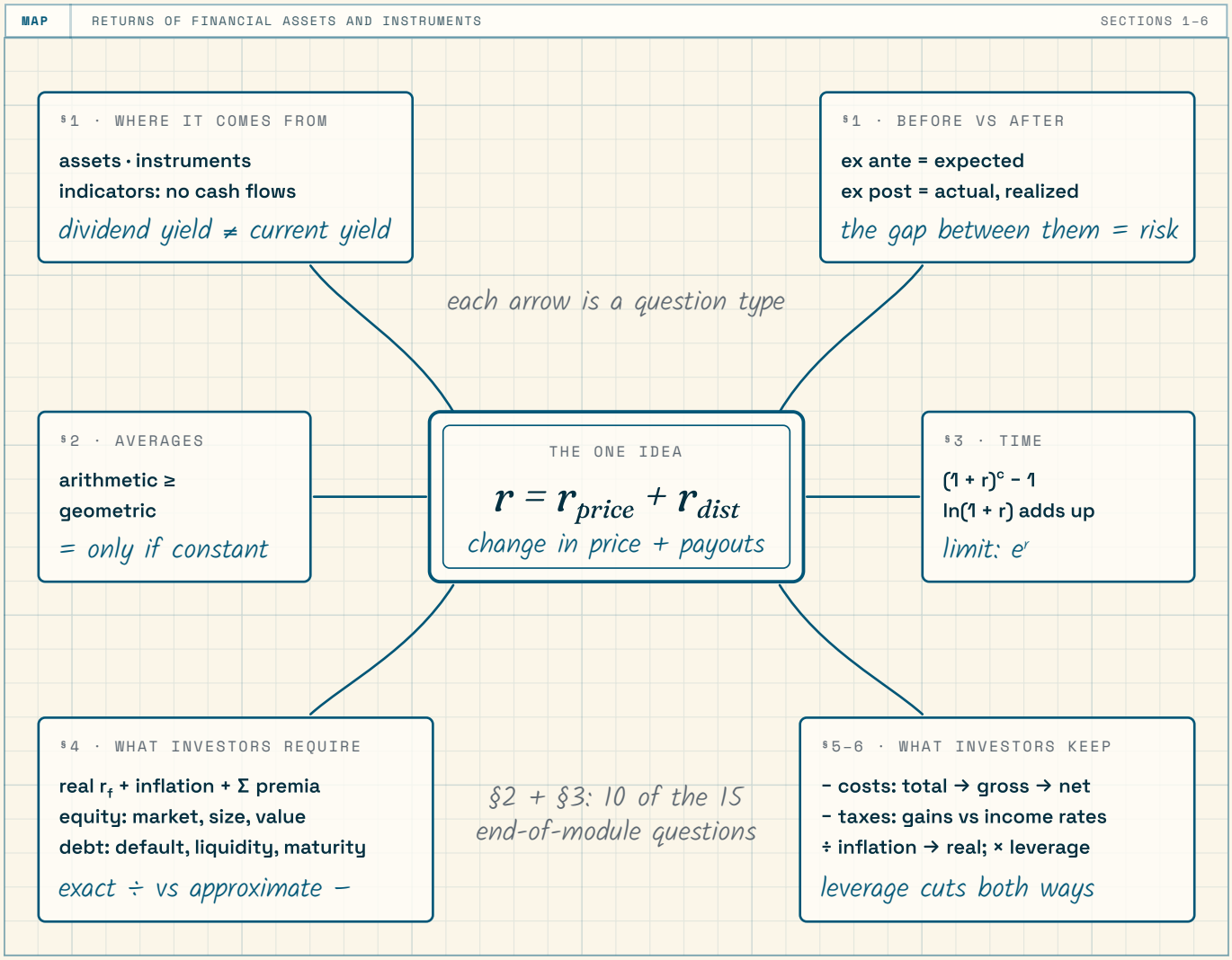
Contents

1	Where returns come from	04
2	Averaging returns over several periods	07
3	Time: annualizing and continuous compounding	09
4	Required return, risk-free rate and risk premia	12
5	Real, net and after-tax returns	16
6	Leveraged returns	18
T	Toolkit: formulas, definitions, relations, calculator	20
P	Practice questions and worked answers	25

OVERVIEW

The module on one page

One idea, the total return, seen from six angles. If you can redraw this page from memory, you know the module.



5 Things to remember

1. Total return = price return + distribution return, both over the **starting** price.
2. Geometric mean ≤ arithmetic mean. The gap grows with volatility.
3. Annualize by compounding: $(1 + r)^c - 1$. Never multiply.
4. Log returns add. Simple returns compound.
5. Exact adjustments divide: $(1 + r)/(1 + x) - 1$. Subtracting is the shortcut.

WHERE THE WEIGHT SITS

"Geometric mean" appears 31 times in the reading. The tax section is about 15% of the prose. And 10 of the 15 end-of-module questions sit in §2 and §3.

§ 1 · WHERE RETURNS COME FROM

What earns a return?

Before any formula, be clear about what you are measuring. The curriculum sorts everything with a return into three groups.

FINANCIAL ASSETS
Cash, **equity** (ownership), **debt** (the lender's claim on the borrower) and hybrids that mix the two.

FINANCIAL INSTRUMENTS
Standardized, tradable contracts that package cash flows: shares, bonds, rights, derivatives.

FINANCIAL INDICATORS
Observable but **not directly tradable**: exchange rates, interest rates, market indexes.

instrument = how an asset is packaged and traded

Shares and bonds belong to both groups: they are assets, standardized enough to trade as instruments. Instruments are the wrapping that lets assets change hands efficiently. Indicators generate no cash flows of their own. You can measure the return of an index from its levels, but nobody pays you for holding it.

1.1 Expected versus actual

decide on ex ante, live with ex post

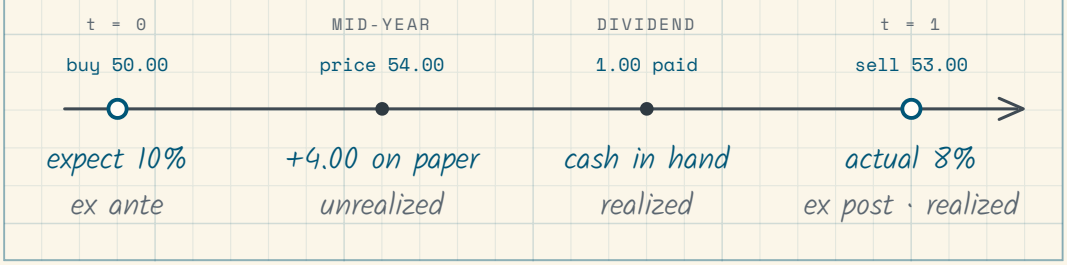
An ex ante (expected) return is a forecast made before the outcome is known, built from history, current market conditions and analysis. An ex post (actual) return is measured after the period has ended. Decisions are made on the first. Results are judged on the second.

The two differ because markets, the economy and the issuer surprise you. The less sure investors are of getting what they expect, the more they ask to be paid for holding on. That payment is the **risk premium** of §4.

1.2 Realized versus unrealized

While you hold an investment, its change in value is an unrealized return, a "paper gain" or loss. It becomes realized when you sell. Dividends and interest are realized when you receive them. Taxes are usually charged only on what is realized (§5).

FIG. 1.1 ONE YEAR OF ONE SHARE (53 − 50 + 1) / 50 = 8%



1.3 Total return: two sources, one denominator

Every return comes from one or both of two places: the price moves, or the investment pays you something.

also called capital appreciation return

EQ. 1 $$r_{\text{price}} = \frac{P_1 - P_0}{P_0}$$ *price return*

dividends, interest, other payouts

EQ. 2 $$r_{\text{dist}} = \frac{Inc_1}{P_0}$$ *capital distribution return*

holding period return (HPR) when measured over the period you held it

EQ. 3 $$r = r_{\text{price}} + r_{\text{dist}} = \frac{P_1 - P_0 + Inc_1}{P_0}$$ *total return*

For a share, the distribution return is the dividend yield: annual dividends over the share price at the start of the period. For a bond it is the current yield: annual coupons over the bond's market price (next page). Returns may be written as a decimal (0.125), a fraction or a percentage (12.5%). They are the same number.

both parts over P_0, the price you paid

WORKED 1.1 A share, one year

given bought at 48.00 · dividend 1.20 during the year · price 52.80 at year end

PRICE $= \frac{52.80 - 48.00}{48.00} = \frac{4.80}{48.00} = 10.00\%$

DIVIDEND YIELD $= \frac{1.20}{48.00} = 2.50\%$

TOTAL $= 10.00\% + 2.50\% = \underline{12.50\%}$ check: 6.00 / 48.00 ✓

ON THE CALCULATOR · 4 DP *read 12.50% · Chn adds before it divides: no brackets*

BA II PLUS	52.8 [+] 1.2 [÷] 48 [=]	12 KEYS	→ 1.1250
HP 12C	52.8 [ENTER] 1.2 [+] 48 [÷]	12 KEYS	→ 1.1250

EXAM TRAP

Two classic wrong answers: dividing by the ending price (6.00 / 52.80 = 11.36%), and leaving out the dividend (10.00%). With a falling price, a third: subtracting the dividend instead of adding it.

TRY IT · 60 SECONDS

A share bought at 30.00 pays a 0.90 dividend and ends the year at 28.20. What is its total return?

answer: −3.00%

1.4 Bonds: current yield is not the coupon rate

The coupon rate divides the annual coupon by **par**. The current yield divides the same coupon by the bond's **market price**. When the price is above par, the current yield is below the coupon rate, and the reverse.

here the coupon only just rescues a falling price

WORKED 1.2 **A premium bond, one year**

given 3.00% annual coupon · par 100 · bought at 104.00 · 101.40 a year later

PRICE $= \frac{101.40 - 104.00}{104.00} = -2.50\%$

CURRENT YIELD $= \frac{3.00}{104.00} = 2.88\%$ ← below the 3.00% coupon: price > par

TOTAL $= -2.50\% + 2.88\% = \mathbf{0.38\%}$ check: 0.40 / 104.00 ✓

ON THE CALCULATOR · 4 DP — read 0.38% · stop at the growth factor

BA II PLUS	101.4 [+] 3 [÷] 104 [=]	12 KEYS	→ **1.0038**
HP 12C	101.4 [ENTER] 3 [+] 104 [÷]	12 KEYS	→ **1.0038**

BOND PRICE VS PAR	CURRENT YIELD VS COUPON RATE	WHY
above par (premium)	**current yield < coupon**	*bigger denominator*
at par	current yield = coupon	*same denominator*
below par (discount)	**current yield > coupon**	*smaller denominator*

1.5 Some investments have one source only

A share that pays no dividend can only gain or lose on price. An **annuity** works the other way round: you receive a fixed series of payments, and that is all.

1.6 Three conventions that save marks

1 + r is the growth factor; r is the return

1. **Compare like with like.** Only compare returns measured over the same period length. Daily, monthly or annual is fine, but not a mix (§3 fixes mixes).
2. **Use closing prices.** An annual return runs from the close of the last trading day of one year to the close of the last trading day of the next. Monthly returns go month-end close to month-end close. Each market uses its own last trading day.
3. **Check the notation.** Some authors write R for 1 + r, others the other way round. Before you plug a symbol in, ask what is being measured, and how.

§ 2 · ACROSS SEVERAL PERIODS

Two averages, two questions

The arithmetic mean answers: what was a typical single-period return? The geometric mean answers: at what steady rate did the money actually grow?

simple average of the period returns

EQ. 4 $$\bar{r} = \frac{r_1 + r_2 + \ldots + r_T}{T} = \frac{1}{T}\sum r_t$$ *arithmetic mean*

compound average: returns earned on returns

EQ. 5 $$r_G = [(1 + r_1)(1 + r_2)\ldots(1 + r_T)]^{1/T} - 1$$ *geometric mean*

The product inside the brackets, minus 1, is the holding period return for the whole span. The geometric mean is the single rate that, compounded T times, produces exactly that growth.

100 → 125 → 100. You made nothing.

WORKED 2.1 Up 25%, then down 20%

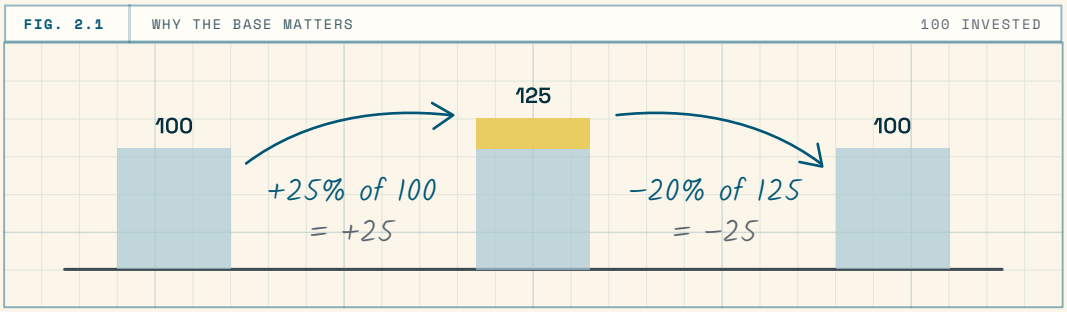
ARITHMETIC $= \frac{25\% + (-20\%)}{2} = 2.50\%$ ← *looks like a gain*

GEOMETRIC $= (1.25 \times 0.80)^{1/2} - 1 = 1.00^{1/2} - 1 = \mathbf{0.00\%}$ ✓

ON THE CALCULATOR · 4 DP *read 0.00% · two periods: √x replaces y^x ½*

BA II PLUS	1.25 [×] .8 [=] [√x]	9 KEYS	→ 1.0000
HP 12C	1.25 [ENTER] .8 [×] [g] [√x]	10 KEYS	→ 1.0000

FIG. 2.1 WHY THE BASE MATTERS — 100 INVESTED



WHY IT MATTERS

The arithmetic mean quietly assumes the amount invested never changes. After every gain or loss it does. To describe growth over time, use the geometric mean.

TRY IT · 60 SECONDS

Up 40%, then down 30%. Find the arithmetic and the geometric mean return.

answer: 5.00% and −1.01%

add, then divide

WORKED 2.2 Five years, both averages

given annual total returns: +18% · −12% · +9% · +24% · −6% · 10,000 invested at the start

ARITHMETIC $= \frac{18 - 12 + 9 + 24 - 6}{5} = \frac{33}{5} = 6.60\%$

GROWTH FACTOR $= 1.18 \times 0.88 \times 1.09 \times 1.24 \times 0.94 = 1.31929$

5-YEAR HPR $= 1.31929 - 1 = 31.93\%$ ← *the whole span, not a yearly rate*

GEOMETRIC $= 1.31929^{1/5} - 1 = \mathbf{5.70\%}$

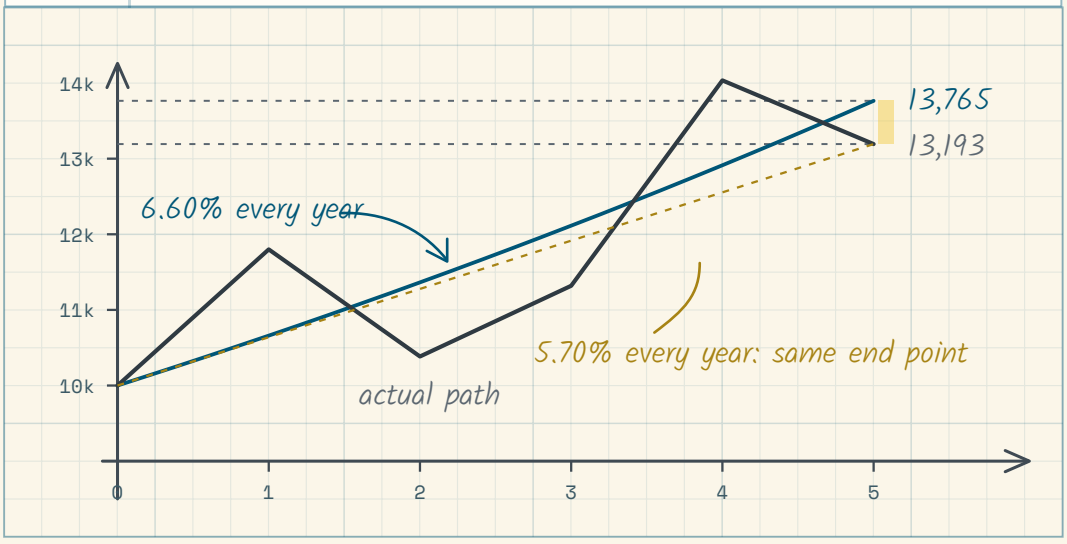
CHECK $10{,}000 \times 1.05698^{5} = 13{,}193 =$ the account ✓

ON THE CALCULATOR · 4 DP *read 5.70% · no = before y^x, no − 1 at the end*

BA II PLUS: 1.18 [×] .88 [×] 1.09 [×] 1.24 [×] .94 [y^x] 5 [1/x] [=] — 26 KEYS → **1.0570**

HP 12C: 1.18 [ENTER] .88 [×] 1.09 [×] 1.24 [×] .94 [×] 5 [1/x] [y^x] — 26 KEYS → **1.0570**

FIG. 2.2 SAME AVERAGE, LESS MONEY — SAME DATA AS THE COVER



the arithmetic-geometric mean inequality

2.1 The rule the exam loves

- Geometric ≤ arithmetic, always. They are equal only when every period's return is identical.
- The more the returns vary, the wider the gap. Volatility costs compound growth.
- Both are usually reported. The arithmetic mean describes the typical period. The geometric mean describes growth: for the investor in Worked 2.2, 31.93% in total, or 5.70% a year.
- **Trap:** dividing the total HPR by T (31.93 / 5 = 6.39%) gives neither mean.

§ 3 · TIME

Put every return on a year

A 2.4% return over 13 weeks and a 20.8% return over two years can't be compared as they stand. Convert both to an annual rate first.

c = periods per year

EQ. 6 $r_{ann} = (1 + r_{period})^{c} - 1$ *compound, never multiply*

c < 1 when the period is longer than a year

WORKED 3.1 Three offers on your desk

OFFER	PERIOD	RETURN	C	ANNUALIZED	RANK
A	13 weeks	2.40%	52/13 = 4	9.95%	2nd
B	1 year	10.10%	1	**10.10%**	1st
C	2 years	20.80%	12/24 = 0.5	9.91%	3rd

OFFER A $= 1.024^{4} - 1 = 9.95\%$ ← *a small number, a fair rate*

OFFER C $= 1.208^{0.5} - 1 =$ 9.91% ← *biggest raw number, last place*

ON THE CALCULATOR · 4 DP *spot the exponent: 4 = square twice, ½ = one square root*

A · BA II PLUS	1.024 x² x²	7 KEYS	→ **1.0995**
A · HP 12C	1.024 ENTER 4 yˣ	8 KEYS	→ **1.0995**
C · BA II PLUS	1.208 √x	6 KEYS	→ **1.0991**
C · HP 12C	1.208 g √x	7 KEYS	→ **1.0991**

3.1 Going the other way

not 8% / 12 = 0.667%

To turn an annual rate into a monthly or quarterly one, rearrange Eq. 6: $r_{period} = (1 + r_{ann})^{1/c} - 1$. An 8% annual rate is $1.08^{1/12} - 1 = 0.643\%$ a month.

3.2 What annualizing assumes

Annualizing assumes the period's return could be earned **again and again**, with the money reinvested at the same rate. That is rarely possible, which is why the standard disclaimer warns that past performance does not indicate future results. Applied to periods shorter than a year, it also stretches a short burst of performance over twelve months and ignores how variable returns are. It can badly overstate the annual figure.

SEE IT

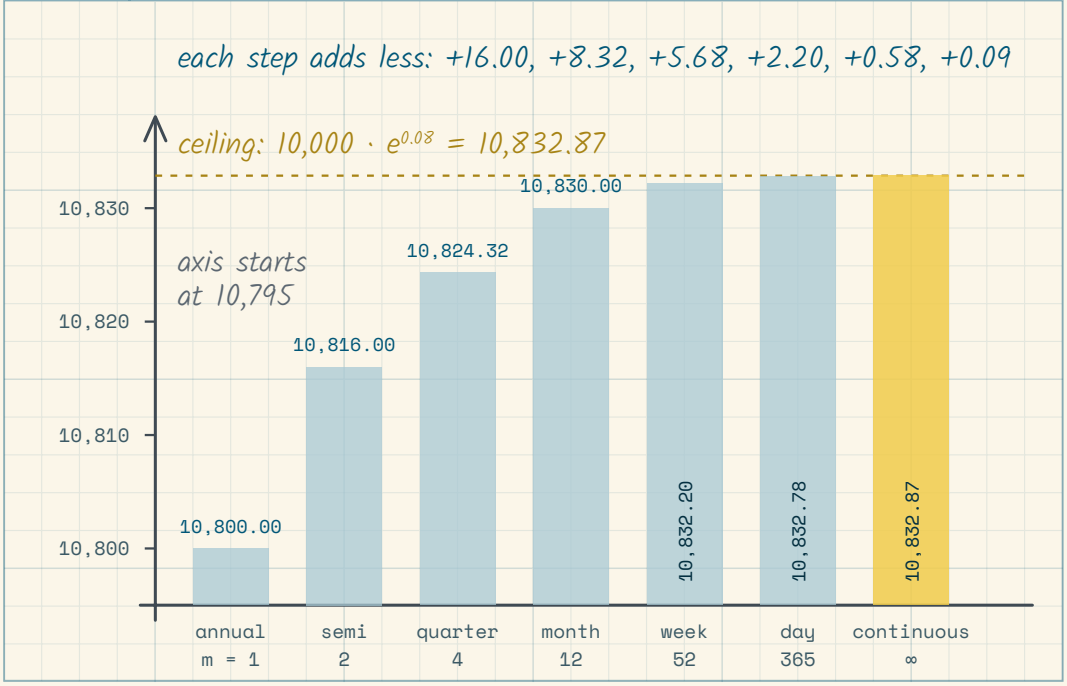
A stock that rises 0.8% a day for 250 trading days: $1.008^{250} - 1 \approx +633\%$. Falling 0.8% a day: $0.992^{250} - 1 \approx -86.6\%$. Losses can never pass −100%. Gains have no ceiling.

§ 3 · TIME

3.3 Compound more often: you gain, but less each time

Hold the stated annual rate fixed and compound more often: annually, then semiannually, quarterly, monthly, weekly, daily. You end with a little more each time, but the extra shrinks toward a ceiling. That ceiling is continuous compounding, growth of e^{rt}, where $e \approx 2.71828$.

FIG. 3.1 | 10,000 AT A STATED 8% FOR ONE YEAR | M COMPOUNDING PERIODS



log return = continuously compounded return

EQ. 7 $$r_{cc} = \ln\left(\frac{P_{t+1}}{P_t}\right) = \ln(1 + r)$$ and back: $r = e^{r_{cc}} - 1$

Here r is the simple return, r_{cc} the log return. Other books use other letters.

logs sit below simple returns: less up, more down

WORKED 3.2 Simple vs log, both directions

40.00 → 46.00 $r = 15.00\%$ · $r_{cc} = \ln(46/40) = \ln 1.15 = \underline{13.98\%}$

46.00 → 41.40 $r = -10.00\%$ · $r_{cc} = \ln(41.40/46) = \ln 0.90 = \underline{-10.54\%}$

ON THE CALCULATOR · 4 DP read 13.98%

BA II PLUS	46 ÷ 40 = LN	7 KEYS	→ 0.1398
HP 12C	46 ENTER 40 ÷ g LN	8 KEYS	→ 0.1398

TRY IT · 60 SECONDS

A price falls from 50.00 to 45.50. Give the simple and the log return.

answer: −9.00% and −9.43%

§ 3 · TIME

3.4 The superpower: log returns add up

The return over many periods is a *product* of price ratios: $P_T/P_0 = (P_1/P_0)\times(P_2/P_1)\times \dots \times(P_T/P_{T-1})$. Take natural logs and the product turns into a *sum*:

the additive property: the reason analysts love logs

EQ. 8 $$r_{cc}(0,T) = r_{cc}(0,1) + r_{cc}(1,2) + \dots + r_{cc}(T-1,T) = \sum r_{cc}(t-1,t)$$

simple returns don't add: 11.28% is not the answer

WORKED 3.3 Three months, one quarter

MONTH END	PRICE	SIMPLE R	LOG $R_{cc} = \ln(P_T/P_{T-1})$
start	40.00	–	–
month 1	46.00	+15.00%	+13.976%
month 2	41.40	-10.00%	-10.536%
month 3	44.00	+6.28%	+6.091%
sum		11.28% ✗	**9.531%**

ADD THE LOGS $13.976 - 10.536 + 6.091 = 9.531\%$

CHECK DIRECTLY $\ln(44.00 / 40.00) = \ln 1.10 = 9.531\%$ ✓

BACK TO SIMPLE $e^{0.09531} - 1 =$ 10.00% ← *the real quarter: 40 → 44*

ON THE CALCULATOR · 4 DP *read 9.53% · one division beats three logs*

BA II PLUS	44 ÷ 40 = LN	7 KEYS	→ **0.0953**
HP 12C	44 ENTER 40 ÷ g LN	8 KEYS	→ **0.0953**

3.5 When to reach for which

YOU NEED TO…	SIMPLE RETURNS	LOG RETURNS
chain periods together	multiply (1 + r)	**add r_{cc}**
report what an investor earned	**yes**, that's the HPR	convert back: $e^{r_{cc}} - 1$
go from a period log return to the full span	—	sum the periods

EXAM TRAP

Two half-years with log returns of 4.2% and −1.6% make 2.6% for the year, as a log return. The year's simple return is $e^{0.026} - 1 = 2.63\%$. Compounding the two as if they were simple ($1.042 \times 0.984 - 1 = 2.53\%$) is wrong.

TRY IT · 60 SECONDS

Monthly log returns of 2.0%, −3.5% and 4.0%. What is the quarter's simple return?

answer: $e^{0.025} - 1 =$ 2.53%

§ 4 · WHAT INVESTORS REQUIRE

The price of waiting

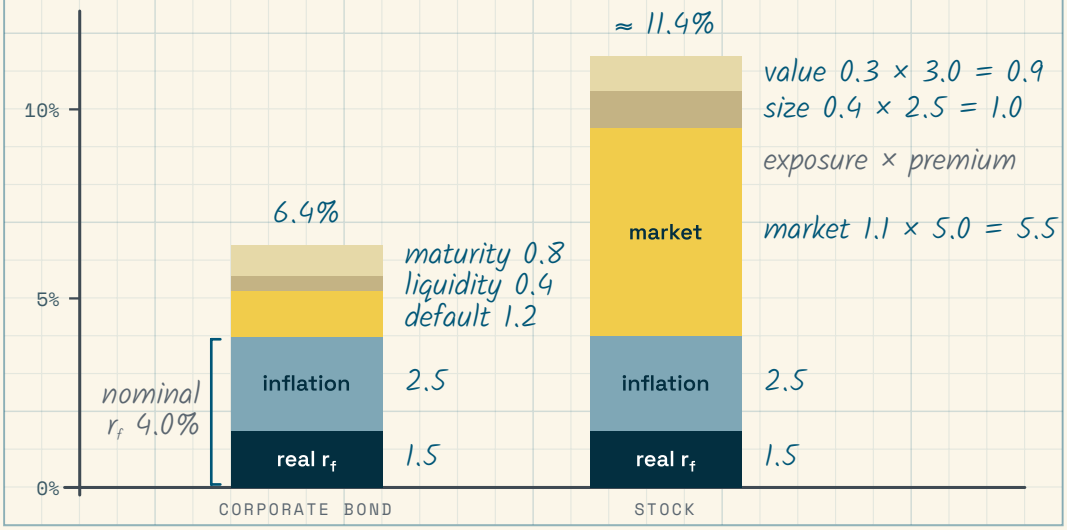
Money you invest is money you don't spend today. The required rate of return is what it takes to make that wait worth it.

opportunity cost: the value of the choice you didn't make

Investing means delaying consumption, and that delay has an opportunity cost. The required rate of return is the lowest return at which an investor will still commit the money. Anyone raising capital, a company selling shares or bonds or a government borrowing, has to clear that bar or go without.

To compare investments, analysts break the required return into building blocks: a risk-free rate, plus **risk premia** that each pay for one kind of risk. *Risk* here means uncertainty: the chance that the actual return differs from the expected one.

FIG. 4.1 EVERY RATE IS A STACK — 1 SQUARE = 1% · ILLUSTRATIVE



4.1 The risk-free rate

no default risk, no reinvestment risk

The risk-free rate is what you earn on an asset that cannot default and leaves you no reinvestment risk. In practice analysts use short-term government bills, such as the 30-day US Treasury bill. Their short maturity makes them relatively insensitive to interest-rate changes. A government can raise taxes to repay, which keeps default risk minimal. This nominal rate has two parts: the **real risk-free rate**, which reflects time preference (how much people prefer consuming now rather than later), and **expected inflation**.

§ 4 · WHAT INVESTORS REQUIRE

4.2 Taking inflation out: exact or quick

The real risk-free rate is the *geometric* difference between the nominal risk-free rate and inflation. The quick version subtracts.

exact: divide growth factors

EQ. 9 $$r_{rf} = \frac{1 + r_f}{1 + inf} - 1$$ *real risk-free rate*

quick: subtract rates

EQ. 10 $$r_{rf} \approx r_f - inf$$ *fine when rates are small, or continuously compounded*

the shortcut overstates whenever r_f > inflation > 0

WORKED 4.1 **Nominal risk-free rate 4.50%, inflation 3.00%**

EXACT $= \frac{1.045}{1.030} - 1 = 1.46\%$

QUICK $= 4.50\% - 3.00\% = 1.50\%$ ← 0.04 points high

ON THE CALCULATOR · 4 DP *read 1.46%*

BA II PLUS	1.045 ÷ 1.03 =	11 KEYS	→ **1.0146**
HP 12C	1.045 ENTER 1.03 ÷	11 KEYS	→ **1.0146**

4.3 Ex ante versus ex post, in real terms

A fund can beat its nominal target and still lose purchasing power.

beat the target, lost purchasing power

WORKED 4.2 **A bond fund: aimed for 6.0% with 2% inflation, earned 6.5% with 8%**

EXPECTED REAL $= \frac{1.060}{1.02} - 1 = 3.92\%$

REALIZED REAL $= \frac{1.065}{1.08} - 1 = -1.39\%$ ← nominal beat, real loss

ON THE CALCULATOR · 4 DP *read −1.39% · below 1 means a loss*

BA II PLUS	1.065 ÷ 1.08 =	11 KEYS	→ **0.9861**
HP 12C	1.065 ENTER 1.08 ÷	11 KEYS	→ **0.9861**

4.4 Small errors compound

bigger rates, longer horizon, bigger error. A miss in the inflation forecast is often larger still

With a 5% nominal rate and 3% inflation, the exact real rate is 1.94% and the shortcut says 2.00%. One year apart, that is nothing. Compounded, it is not:

HORIZON	EXACT: 1.0194^N	QUICK: 1.0200^N	OVERSTATED BY
1 year	1.0194	1.0200	0.06% of capital
10 years	1.2121	1.2190	0.69% of capital
30 years	1.7806	1.8114	**3.08% of capital**

4 REQUIRED

§ 4 · WHAT INVESTORS REQUIRE

4.5 Risk premia: one for each risk you carry

A risk premium is the extra return expected for bearing a particular risk. How big it is depends on what you hold, for how long, and which risks come with it. The wider the possible gap between what you expect and what you get, the bigger the premium.

≈ : an additive approximation, not an exact compounding identity

EQ. 11 $$r_i \approx r_{\text{rf}} + inf + \sum_{j=1}^{n} f_{ij} \times rp_j$$

where f_{ij} = exposure of security i to factor j · rp_j = premium for factor j

EQUITY PREMIA

- **Market**: the reward for owning shares at all, rather than the risk-free asset.
- **Size**: small companies' extra return over large ones.
- **Value**: cheap stocks' (high book-to-market) extra return over expensive ones.

BOND PREMIA

- **Default**: lending to a weaker borrower than a government. Grows as credit quality falls.
- **Liquidity**: holding a bond you may not be able to sell fast, or only at a lower price.
- **Maturity**: tying money up longer, in bonds whose prices swing more with rates.

exposures multiply the premia; Fig. 4.1 drew this stack

WORKED 4.3 Build a stock's required return

BLOCK	EXPOSURE F	PREMIUM RP	CONTRIBUTION
real risk-free rate	–	–	1.5%
expected inflation	–	–	2.5%
market	1.1	5.0%	5.5%
size	0.4	2.5%	1.0%
value	0.3	3.0%	0.9%
required return			**≈ 11.4%**

premia come and go: not constants

Premia and the real risk-free rate both vary across investments and over time. They move with economic fundamentals and with expectations about how those fundamentals will change. A premium can disappear for years and come back. A security's exposures can change too.

EXAM TRAP

A risk premium does not guarantee that actual returns will meet expected returns. It pays for the uncertainty. The real risk-free rate pays for time, not risk.

§ 4 · WHAT INVESTORS REQUIRE

4.6 Excess returns: beating a reference

An excess return measures performance against a reference rate or benchmark: the risk-free rate, the real risk-free rate, or another benchmark such as a market index. As with inflation, there is an exact version and a quick one.

relative wealth: how much richer than the benchmark?

EQ. 12 $$r_{\text{ex}} = \frac{1 + r_i}{1 + r_b} - 1$$ *exact excess return*

EQ. 13 $$r_{\text{ex}} \approx r_i - r_b$$ *arithmetic approximation*

same fund, two references

WORKED 4.4 A fund that returned 11.0%

given fund 11.0% · Treasury bill 4.0% · market index 9.5%

VS BILL · EXACT $= \frac{1.110}{1.040} - 1 =$ **6.73%** *quick: 7.00%*

VS INDEX · EXACT $= \frac{1.110}{1.095} - 1 =$ **1.37%** *quick: 1.50%*

ON THE CALCULATOR · 4 DP *read 6.73%*

BA II PLUS	1.11 ÷ 1.04 =	10 KEYS	→ 1.0673
HP 12C	1.11 ENTER 1.04 ÷	10 KEYS	→ 1.0673

A SHORTCUT TO THE EXACT ANSWER

Exact = approximate ÷ (1 + benchmark), because $(1 + r)/(1 + b) - 1 = (r - b)/(1 + b)$. Check: *7.00 / 1.04 = 6.73* ✓. The same trick works for real returns: $(r - inf)/(1 + inf)$.

4.7 Excess return is not a risk premium

after vs before, again

A realized excess return is **ex post**: what a fund actually earned over its benchmark. A risk premium is **ex ante**: the extra return investors *expect* for bearing a risk. A fund can show a negative excess return in a year when the equity risk premium, as an expectation, is perfectly positive. Read questions carefully for "expected" versus "realized".

TRY IT · 60 SECONDS

A portfolio returns 5.0% while the risk-free rate is 2.0%. Find the exact excess return.

answer: 1.05/1.02 − 1 = 2.94%

§ 5 · WHAT INVESTORS KEEP

From total return to what you keep

Inflation, costs and taxes each take a slice. Measure them separately: the exam asks about each one.

5.1 Real returns

same shape as Eq. 9, with the actual return on top

EQ. 14 $$r_{real} = \frac{1 + r}{1 + inf} - 1$$

compare across years with different inflation

dividend assumed paid at year end, not reinvested

WORKED 5.1 A losing year, then inflation on top

given bought at 62.00 · dividend 1.86 · sold at 55.80 · inflation 4.5%

NOMINAL $= \frac{55.80 - 62.00 + 1.86}{62.00} = \frac{-4.34}{62.00} = -7.00\%$

REAL $= \frac{0.93}{1.045} - 1 = \underline{\underline{-11.00\%}}$ ← *purchasing power lost*

ON THE CALCULATOR · 4 DP *read −11.00% · nominal and real in one pass*

BA II PLUS	55.8 [+] 1.86 [÷] 62 [÷] 1.045 [=]	19 KEYS	→ 0.8900
HP 12C	55.8 [ENTER] 1.86 [+] 62 [÷] 1.045 [÷]	19 KEYS	→ 0.8900

5.2 Gross and net returns

gross return: judge the manager

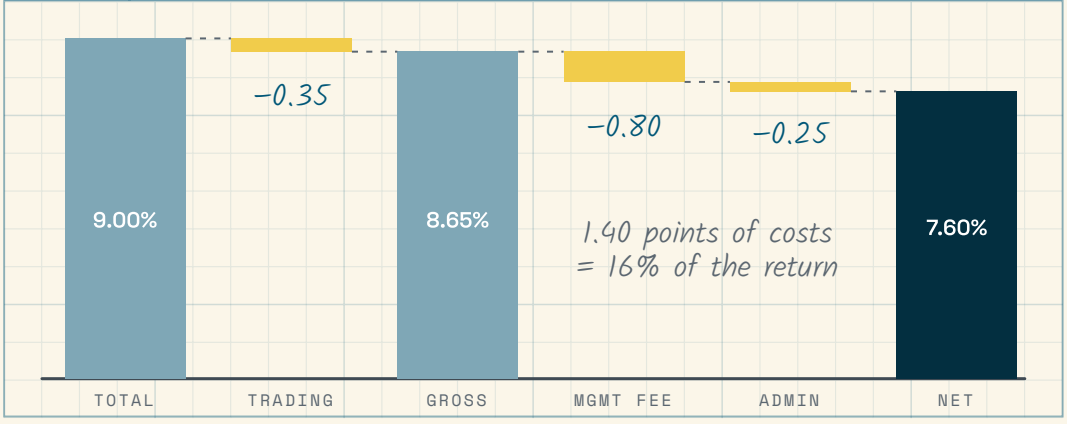
Gross return is the manager's result once the costs of executing the strategy, commissions and other trading costs, are paid. Running costs are left out, so it is the fair yardstick for comparing managers. Net return then takes off the management fee and administrative expenses, such as custody and performance fees. It is what reaches the investor.

net return: judge the deal

EQ. 15

total return − transaction costs = gross return
gross return − management fees − administrative expenses = net return

FIG. 5.1 COSTS, ONE SLICE AT A TIME — 1 SQUARE = 1%



5 ADJUSTED

§ 5 · WHAT INVESTORS KEEP

5.3 After-tax returns: two incomes, two rates

Tax systems usually tax realized capital gains and distributions (dividends, interest) as income, often at **different rates.** Long-term gains frequently get a lower rate than short-term ones, and capital losses can usually offset gains. So pre-tax returns overstate what a taxable investor keeps. The after-tax nominal return deducts each tax from its own source.

tax only on a positive, realized price return

EQ. 16 $$\text{capital gains tax} = (P_1 - P_0) \times t_{cg}$$

EQ. 17 $$r_{\text{after-tax}} = r_{\text{price}}(1 - t_{cg}) + r_{\text{dist}}(1 - t_{\text{dist}})$$

each source keeps its own rate

WORKED 5.2 **Bought, paid, sold within the year**

given bought at 80.00 · dividend 2.40 · sold at 92.00 · gains taxed 20% · dividends 30%

PRICE RETURN = 12.00 / 80.00 = 15.00%

DIVIDEND YIELD = 2.40 / 80.00 = 3.00%

AFTER TAX = 15 × 0.80 + 3 × 0.70 = 12.00 + 2.10 = **14.10%**

TAX BILL 18.00% − 14.10% = 3.90 points ← 22% of pre-tax

ON THE CALCULATOR · 4 DP *read 14.10% · = closes the bracket for you*

BA II PLUS	15 × .8 + (3 × .7 =	12 KEYS	→ **14.1000**
HP 12C	15 ENTER .8 × 3 ENTER .7 × +	12 KEYS	→ **14.1000**

EXAM TRAP

Swap the rates and you get 15 × 0.70 + 3 × 0.80 = 12.90%. Apply one rate to both and you get 18 × 0.70 = 12.60%. Both turn up as answer choices.

5.4 Unrealized gains, real after-tax returns

holding on defers the gains tax

Taxes usually fall on *realized* gains. Had the investor in Worked 5.2 kept the shares, this year's tax would fall on the dividend alone: 2.40 × 30% = 0.72 a share, just 0.90% of the price paid. What finally pays for waiting and for risk is the **after-tax real return:** *1.141 / 1.03 − 1 = 10.78%* with 3% inflation, against 11.10% by the quick method.

Managers rarely report it, because it depends on the investor: their marginal tax rate, the holding period, the account type (tax-exempt, tax-deferred or taxable) and every jurisdiction that taxes them. Taxable investors cut taxes by choosing tax-efficient investments, trading less and delaying gains. Delay may also qualify for a lower long-term rate, and the deferred tax keeps earning returns.

5 ADJUSTED

§ 6 · LEVERAGE

Borrowed money cuts both ways

Leverage lets you invest more than your own capital, by borrowing or through derivatives. It magnifies returns and, with them, risk. Buying shares on margin and buying property with a mortgage are everyday examples.

V_E = your equity · V_D = debt · r_D = borrowing cost

leverage helps only while $r_p > r_D$

EQ. 18 $$r_{LP} = \frac{r_p(V_E + V_D) - V_D r_D}{V_E} = r_p + \frac{V_D}{V_E}(r_p - r_D)$$

WORKED 6.1 2,000,000 portfolio: 1,200,000 own money, 800,000 borrowed at 6%

RATIO $V_D / V_E = 800{,}000 / 1{,}200{,}000 = 0.667$

GOOD YEAR · 10% $= 10\% + 0.667 \times (10\% - 6\%) = \mathbf{12.67\%}$

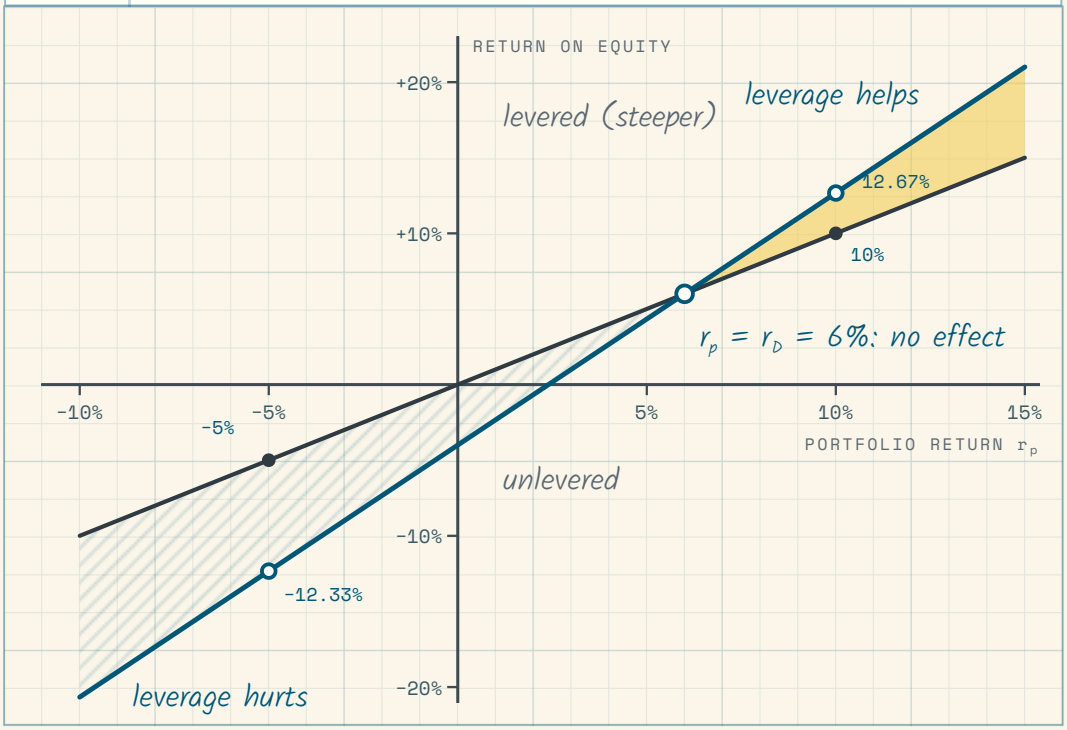
IN CASH $(200{,}000 - 48{,}000 \text{ interest}) / 1{,}200{,}000 = 12.67\%$ ✓

BAD YEAR · -5% $= -5\% + 0.667 \times (-5\% - 6\%) = \mathbf{-12.33\%}$

ON THE CALCULATOR · 4 DP — read 12.67% · 800/1,200 = 2/3 and 10 − 6 = 4 in your head

BA II PLUS	2 ÷ 3 × 4 + 10 =	9 KEYS	→ **12.6667**
HP 12C	2 ENTER 3 ÷ 4 × 10 +	9 KEYS	→ **12.6667**

FIG. 6.1 RETURN ON YOUR EQUITY VS PORTFOLIO RETURN — V_D/V_E = 0.667 · R_D = 6%



§ 6 · LEVERAGE

6.1 Leverage without borrowing: a call option

An option's value comes entirely from the asset it is written on. A call option gives its buyer the right, but not the obligation, to buy the underlying at a fixed **strike price** before the option expires. The buyer pays a **premium** for that right. The option is worth exercising only when the stock price is above the strike.

payoff = max(S − X, 0) at expiry

EQ. 19 $$r_{\text{call}} = \frac{\text{payoff} - \text{premium}}{\text{premium}}$$

return on the money you put in

1,000 buys 20 shares, or calls on 400 shares

WORKED 6.2 **Same 1,000, two bets**

given stock at 50.00 · 6-month call, strike 52.00 · premium 2.50 per share

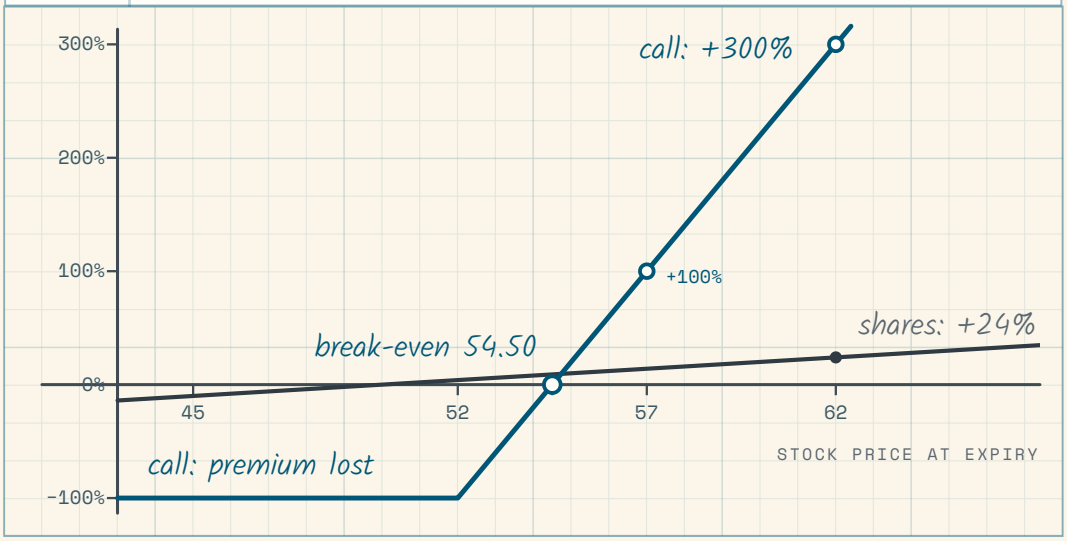
STOCK AT EXPIRY	STOCK RETURN	CALL PAYOFF	CALL RETURN	NOTE
45.00	-10%	0.00	-100%	*premium lost*
52.00	+4%	0.00	-100%	*stock up, call still worthless*
54.50	+9%	2.50	0%	*break-even: X + premium*
57.00	+14%	5.00	+100%	
62.00	+24%	10.00	**+300%**	*12.5× the stock's return*

AT 62.00 $$\frac{(62 - 52) - 2.50}{2.50} = \frac{7.50}{2.50} = +300\%$$ *shares: +24%*

ON THE CALCULATOR · 4 DP *read +300% · payoff 10 ÷ premium, minus 1*

BA II PLUS	10 ÷ 2.5 − 1 =	9 KEYS	→ 3.0000
HP 12C	10 ENTER 2.5 ÷ 1 −	9 KEYS	→ 3.0000

FIG. 6.2 SAME VIEW, TWO BETS — RETURN ON MONEY INVESTED



Options magnify gains and losses much like borrowing does, but asymmetrically: the most you can lose is the premium, and gains begin only once the stock passes the strike plus the premium.

TOOLKIT · FORMULAS

19 formulas

Formulas

EQ. 1 Price return — §1.3

$$r_{\text{price}} = \frac{P_1 - P_0}{P_0}$$

EQ. 2 Capital distribution return — §1.3

$$r_{\text{dist}} = \frac{Inc_1}{P_0}$$

EQ. 3 Total (holding period) return — §1.3

$$r = \frac{P_1 - P_0 + Inc_1}{P_0}$$

EQ. 4 Arithmetic mean — §2

$$\bar{r} = \frac{1}{T}\sum r_t$$

EQ. 5 Geometric mean — §2

$$r_G = [(1 + r_1)...(1 + r_T)]^{1/T} - 1$$

EQ. 6 Annualized return — §3

$$r_{\text{ann}} = (1 + r_{\text{period}})^c - 1$$

EQ. 7 Continuously compounded return — §3.3

$$r_{\text{cc}} = \ln(P_1/P_0) = \ln(1 + r); \quad r = e^{r_{cc}} - 1$$

EQ. 8 Log returns add — §3.4

$$r_{\text{cc}}(0,T) = \sum r_{\text{cc}}(t-1,t)$$

EQ. 9 Real risk-free rate (exact) — §4.2

$$r_{\text{rf}} = \frac{1 + r_{\text{f}}}{1 + inf} - 1$$

EQ. 10 Real risk-free rate (quick) — §4.2

$$r_{\text{rf}} \approx r_{\text{f}} - inf$$

EQ. 11 Required return, built up — §4.5

$$r_i \approx r_{\text{rf}} + inf + \sum f_{ij} \times rp_j$$

EQ. 12 Excess return (exact) — §4.6

$$r_{\text{ex}} = \frac{1 + r_i}{1 + r_b} - 1$$

EQ. 13 Excess return (quick) — §4.6

$$r_{\text{ex}} \approx r_i - r_b$$

EQ. 14 Real return — §5.1

$$r_{\text{real}} = \frac{1 + r}{1 + inf} - 1$$

EQ. 15 Gross and net — §5.2

total − trading costs = gross

gross − mgmt fees − admin = net

EQ. 16 Capital gains tax — §5.3

$$(P_1 - P_0) \times t_{\text{cg}}$$

EQ. 17 After-tax return — §5.3

$$r_{\text{price}}(1 - t_{\text{cg}}) + r_{\text{dist}}(1 - t_{\text{dist}})$$

EQ. 18 Leveraged return — §6

$$r_{\text{LP}} = r_{\text{p}} + \frac{V_{\text{D}}}{V_{\text{E}}}(r_{\text{p}} - r_{\text{D}})$$

EQ. 19 Return on a bought call — §6.1

$$r_{\text{call}} = \frac{\text{payoff} - \text{premium}}{\text{premium}}$$

Boxed here = boxed in the notes: the twelve to know cold. The other seven are quick to rebuild.

TOOLKIT

TOOLKIT · DEFINITIONS

Say it precisely

Definitions

Actual (ex post) return — The return measured after the period has ended.

After-tax nominal return — Total return minus taxes on realized gains and on distributions, each at its own rate.

Annualized return — A return restated as a one-year rate by compounding: $(1 + r)^c - 1$.

Arithmetic mean return — The simple average of period returns: a typical period, not growth.

Call option — The right, not the obligation, to buy the underlying at the strike price before expiry.

Capital distribution return — Income received (dividends, interest) divided by the initial price.

Continuously compounded (log) return — $r_{cc} = \ln(1 + r) = \ln(P_1/P_0)$. Additive over time.

Current yield — Annual coupon divided by the bond's market price.

Default risk premium — Extra return for a riskier issuer's bond instead of a government bond.

Dividend yield — Annual dividends divided by the share price at the start of the period.

Excess return — Return measured against a reference rate or benchmark; exact form $(1 + r)/(1 + b) - 1$.

Expected (ex ante) return — A forecast made before the outcome is known.

Financial asset — Cash, equity, debt and hybrids of the two.

Financial indicator — Observable but not directly tradable: exchange rates, interest rates, indexes. Generates no cash flows.

Financial instrument — A standardized, tradable contract that packages cash flows: shares, bonds, rights, derivatives.

Geometric mean return — The compound average: the steady rate that reproduces the actual growth.

Gross return — Return after trading costs, before management fees and administrative expenses.

Holding period return — The total return over the whole period an investment is held.

Leverage — Borrowing, or using derivatives, to invest more than your own capital.

Liquidity premium — Extra return for an asset that is hard to sell quickly or without moving its price.

Market risk premium — Extra return for holding the equity market instead of the risk-free asset.

Maturity risk premium — Extra return for longer bonds, which are more sensitive to interest rates.

Net return — Gross return minus management fees and administrative expenses: what the investor earns.

Opportunity cost — The value given up by choosing one course of action over another.

Price (capital appreciation) return — The change in price divided by the initial price.

Real return — Return adjusted for inflation: $(1 + r)/(1 + \text{inf}) - 1$.

Real risk-free rate — Pays for time preference alone: consuming later rather than now.

Realized vs unrealized — Locked in by a sale or received in cash, vs a "paper" gain on a holding.

Required rate of return — The minimum return an investor must receive to accept an investment.

Risk premium — Extra expected return for bearing a specific risk.

Risk-free rate — A return with no default or reinvestment risk; proxied by short-term government bills.

Size and value premia — Return spreads of small over large caps, and of high over low book-to-market stocks.

Total return — Price return plus capital distribution return.

TOOLKIT · RELATIONS

If this moves, that moves

Relations

IF…		…THEN	REMEMBER IT AS
bond price > par	⇒	current yield < coupon rate	*bigger denominator*
returns vary more	⇒	arithmetic − geometric gap ↑	*volatility costs growth*
returns are constant	⇒	geometric = arithmetic	*the only tie*
compounding frequency ↑	⇒	effective return ↑, toward $e^r - 1$	*ever smaller steps*
period shorter than a year	⇒	c > 1: annualizing magnifies it	*and extrapolates it*
a gain / a loss	⇒	log return below simple / more negative	*ln(1 + r) < r*
inflation ↑	⇒	real return ↓	*divide, don't subtract*
inflation positive	⇒	quick subtraction exaggerates the size of the real return	*(r − inf)/(1 + inf)*
uncertainty of the outcome ↑	⇒	risk premium ↑	*pay for the gap*
credit quality ↓ · liquidity ↓ · maturity ↑	⇒	default · liquidity · maturity premium ↑	*one premium per risk*
fees and costs	⇒	total ≥ gross ≥ net	*manager vs investor*
gain not yet sold	⇒	usually no capital gains tax yet	*tax follows realization*
$r_p > r_D$ · $r_p < r_D$	⇒	leverage raises · lowers the equity return	*crosses at r_D*
V_D/V_E ↑	⇒	both gains and losses magnified more	*steeper line*
$S_T \le X$ · $S_T = X$ + premium	⇒	call return −100% · 0%	*break-even*

TWELVE TRAPS, ONE LINE EACH

1. *Dividing by P_1 instead of P_0.*
2. *Dropping, or subtracting, the dividend.*
3. *"Geometric exceeds arithmetic." Never.*
4. *HPR ÷ T presented as an average.*
5. *Annualizing by multiplying, r × c.*
6. *Adding simple returns across periods.*

…CONTINUED

7. *Compounding log returns as if simple.*
8. *Subtracting when the exact figure is asked.*
9. *Multiplying by (1 + inflation).*
10. *Swapping the two tax rates.*
11. *Forgetting the interest cost of leverage.*
12. *Forgetting to subtract the option premium.*

TOOLKIT

TOOLKIT · CALCULATOR

Fewer keys, fewer mistakes

Calculator

Only two models are allowed in the exam room: the **TI BA II Plus** (or Professional) and the **HP 12C** (any edition: Platinum, 25th or 30th anniversary, Prestige). Manuals stay outside; keystroke cards may come in. Every sequence in this guide was checked against both official manuals and counted key by key.

BA II PLUS · SET UP ONCE

4 decimals: [2nd] [FORMAT] 4 [ENTER] [2nd] [QUIT]

Leave the factory **Chn** mode on: every BA sequence here assumes it. To check, press [2nd] [FORMAT] and [↓] four times.

HP 12C · SET UP ONCE

4 decimals: [f] 4

Every HP sequence here is **RPN**. The classic 12C has nothing else; on the Platinum or Prestige, [f] [RPN] switches it on.

Every display in this guide assumes 4 decimals.

10 Speed rules, in order of payoff

1. **Key growth factors, not percentages.** +18% is 1.18; −12% is .88. No [%], no [+] 1.
2. **Stop at the growth factor.** 1.0570 *is* +5.70%; 0.9861 *is* −1.39%. Skipping [−] 1 [=] saves three keys every time.
3. **Drop the zeros.** .88, not 0.88; 52.8, not 52.80.
4. **BA: think left to right.** In Chn, 52.8 [+] 1.2 [÷] 48 [=] is (52.8 + 1.2) ÷ 48. No brackets needed.
5. **BA: let [=] close the bracket.** 15 [×] .8 [+] [(] 3 [×] .7 [=] finishes on its own.
6. **HP: operator last, brackets never.** 15 [ENTER] .8 [×] 3 [ENTER] .7 [×] [+]. The stack keeps every intermediate result.
7. **Spot easy exponents.** c = 4: [x²] [x²]. c = ½: [√x]. Two periods of a geometric mean: [√x] again.
8. **Simplify before you key.** 800,000 / 1,200,000 = 2/3; 10% − 6% = 4. Then 2 [÷] 3 [×] 4 [+] 10 [=].
9. **One chain, two steps.** Nominal and real return together: 55.8 [+] 1.86 [÷] 62 [÷] 1.045 [=].
10. **Never retype a result.** BA: [2nd] [ANS] or [STO]/[RCL]. HP: it is already in the stack.

EXAM TRAP · BA IN CHN MODE

Rule 5 matters. Without the bracket, 15 × .8 + 3 × .7 in Chn gives 10.5; the calculator multiplies (12 + 3) by .7. The after-tax answer is 14.1.

TOOLKIT

TOOLKIT · CALCULATOR

Every calculation, fewest keys

Calculator

The shortest verified sequence for each calculation in this guide, on both calculators. Key counts include every digit and decimal point. Read the answer straight off the display.

TASK	BA II PLUS · CHN	KEYS	HP 12C · RPN	KEYS
Total return read 12.50%	52.8 [+] 1.2 [÷] 48 [=]	12	52.8 [ENTER] 1.2 [+] 48 [÷]	12
Geometric mean read 5.70%	1.18 [×] .88 [×] 1.09 [×] 1.24 [×] .94 [yˣ] 5 [1/x] [=]	26	1.18 [ENTER] .88 [×] 1.09 [×] 1.24 [×] .94 [×] 5 [1/x] [yˣ]	26
Annualize, c = 4 read 9.95%	1.024 [x²] [x²]	7	1.024 [ENTER] 4 [yˣ]	8
Annualize, c = ½ read 9.91%	1.208 [√x]	6	1.208 [g] [√x]	7
De-annualize 8% read 0.64% a month	1.08 [yˣ] 12 [1/x] [=]	9	1.08 [ENTER] 12 [1/x] [yˣ]	9
Log return read 13.98%	46 [÷] 40 [=] [LN]	7	46 [ENTER] 40 [÷] [g] [LN]	8
Log back to simple read 10.00%	.09531 [2nd] [eˣ]	8	.09531 [g] [eˣ]	8
Real risk-free rate read 1.46%	1.045 [÷] 1.03 [=]	11	1.045 [ENTER] 1.03 [÷]	11
Exact excess return read 6.73%	1.11 [÷] 1.04 [=]	10	1.11 [ENTER] 1.04 [÷]	10
Real return, one chain read -11.00%	55.8 [+] 1.86 [÷] 62 [÷] 1.045 [=]	19	55.8 [ENTER] 1.86 [+] 62 [÷] 1.045 [÷]	19
After-tax return read 14.10%	15 [×] .8 [+] [(] 3 [×] .7 [=]	12	15 [ENTER] .8 [×] 3 [ENTER] .7 [×] [+]	12
After-tax real return read 10.78%	1.141 [÷] 1.03 [=]	11	1.141 [ENTER] 1.03 [÷]	11
Leveraged return read 12.67%	2 [÷] 3 [×] 4 [+] 10 [=]	9	2 [ENTER] 3 [÷] 4 [×] 10 [+]	9
Call return read +300%	10 [÷] 2.5 [−] 1 [=]	9	10 [ENTER] 2.5 [÷] 1 [−]	9

WHY THE HP SOMETIMES NEEDS ONE KEY MORE

LN, e^x and $\sqrt{x}$ sit on the blue [g] shift. In exchange, RPN never needs a bracket or an [=].

PRACTICE · 14 QUESTIONS

Your turn

Exam format: three options, one correct. Work in the squares. Answers, worked by hand, start on p. 28.

Q1 · CALCULATION
easy

1 An investor buys a share for 40.00, receives a 1.20 dividend during the year and sells the share at year end for 43.00. The holding period return is *closest to*:

A 7.50% B 9.77% C 10.50%

Q2 · CALCULATION
easy

2 A fund returned +8%, −15% and +11% in three consecutive years. Its holding period return over the three years is *closest to*:

A 0.63% B 1.33% C 1.90%

Q2 · CALCULATION
medium

3 An asset's annual returns were 12%, −9%, 15% and 4%. Its geometric mean return is *closest to*:

A 5.07% B 5.47% C 5.50%

Q3 · CALCULATION
easy

4 A fund earned 2.2% over 10 weeks. Assuming 52 weeks in a year, its annualized return is *closest to*:

A 0.42% B 11.44% C 11.98%

Q3 · CALCULATION
hard

5 A stock has continuously compounded returns of 3.5% in the first half of the year and −1.2% in the second half. Its holding period return for the year is *closest to*:

A 2.26% B 2.30% C 2.33%

PRACTICE

PRACTICE · 14 QUESTIONS

§5 · CALCULATION
easy

6 A portfolio returned 7.5% in a year when inflation was 3.2%. Its real return is *closest to*:

(A) 4.17% (B) 4.30% (C) 10.94%

§4 · CALCULATION
medium

7 A portfolio returned 9.2% while its benchmark returned 6.0%. The excess return, measured as relative wealth, is *closest to*:

(A) 2.93% (B) 3.02% (C) 3.20%

§5 · CALCULATION
easy

8 A fund's total return before costs was 6.40%. Transaction costs were 0.20%, the management fee 0.75% and administrative expenses 0.15%, all as a percentage of assets. Its net return is *closest to*:

(A) 5.30% (B) 5.50% (C) 6.20%

§5 · CALCULATION
medium

9 A share's price return for the year was 9% and its dividend yield 4%. Capital gains are taxed at 15% and dividends at 35%. If the investor sold at year end, the after-tax return is *closest to*:

(A) 8.45% (B) 9.25% (C) 10.25%

§6 · CALCULATION
medium

10 An investor funds a portfolio with 500,000 of their own money and 250,000 borrowed at 5%. If the portfolio returns 3%, the return on the investor's equity is *closest to*:

(A) 2.00% (B) 3.00% (C) 4.00%

PRACTICE

PRACTICE · 14 QUESTIONS

§6 · CALCULATION
medium

11 A stock trades at 120. A call option with a strike price of 120 costs 6 per share. At expiry the stock is at 129. The return on the call investment is *closest to*:

(A) 7.5% (B) 50% (C) 150%

§4 · DEFINITION
easy

12 Which component of a corporate bond's required return compensates the investor for the risk of not being able to sell quickly without a price concession?

(A) Default risk premium
(B) Liquidity premium
(C) Maturity risk premium

§4 · DEFINITION
medium

13 Which component of a required return compensates the investor for postponing consumption, and not for any risk?

(A) The real risk-free rate
(B) The default risk premium
(C) The market risk premium

§5 · REASONING
easy

14 An investor holds a share that rose 20% during the year and paid a 2% dividend, and has not sold it. Under typical tax rules, this year's tax applies to:

(A) the dividend only.
(B) the price gain only.
(C) both the price gain and the dividend.

ANSWER KEY, UPSIDE DOWN

1 C · 2 C · 3 A · 4 C · 5 C · 6 A · 7 B · 8 A · 9 C · 10 A · 11 B · 12 B · 13 A · 14 A

PRACTICE

PRACTICE · WORKED ANSWERS

1 **C** ✓ **10.50%**

(43.00 − 40.00 + 1.20) / 40.00 = 4.20 / 40.00 = **10.50%**

A leaves out the dividend (7.50%). B divides by the selling price, 4.20 / 43.00 = 9.77%.

2 **C** ✓ **1.90%**

1.08 × 0.85 × 1.11 − 1 = 1.01898 − 1 = **1.90%**

A is the geometric mean per year (0.63%). B is the arithmetic mean (1.33%). The question asks for the whole span.

3 **A** ✓ **5.07%**

$(1.12 \times 0.91 \times 1.15 \times 1.04)^{1/4} - 1 = 1.21896^{0.25} - 1 =$ **5.07%**

B divides the HPR by four (21.90 / 4 = 5.47%). C is the arithmetic mean (5.50%), which is always at least as high.

4 **C** ✓ **11.98%**

$c = 52 / 10 = 5.2 \rightarrow 1.022^{5.2} - 1 =$ **11.98%**

B multiplies, 2.2 × 5.2 = 11.44%. A inverts c, $1.022^{10/52} - 1 = 0.42\%$.

5 **C** ✓ **2.33%**

logs add: 3.5% − 1.2% = 2.3% → $e^{0.023} - 1 =$ **2.33%**

B stops at the log return. A compounds the logs as if they were simple returns (1.035 × 0.988 − 1 = 2.26%).

6 **A** ✓ **4.17%**

1.075 / 1.032 − 1 = **4.17%**

B is the subtraction shortcut (4.30%). C multiplies by (1 + inflation): 1.075 × 1.032 − 1 = 10.94%.

7 **B** ✓ **3.02%**

1.092 / 1.060 − 1 = **3.02%** check: 3.20 / 1.06 = 3.02 ✓

C is the arithmetic approximation (3.20%). A divides by the wrong growth factor, 3.20 / 1.092 = 2.93%.

PRACTICE · WORKED ANSWERS

8 **A ✓ 5.30%**

$6.40 - 0.20 - 0.75 - 0.15 = \underline{5.30\%}$

B forgets the transaction costs (5.50%). C is the gross return (6.20%).

9 **C ✓ 10.25%**

$9 \times 0.85 + 4 \times 0.65 = 7.65 + 2.60 = \underline{10.25\%}$

B swaps the rates (9.25%). A taxes everything at 35% (8.45%).

10 **A ✓ 2.00%**

$V_D/V_E = 0.5 \rightarrow 3\% + 0.5 \times (3\% - 5\%) = \underline{2.00\%}$

in cash: $(750{,}000 \times 3\% - 250{,}000 \times 5\%) / 500{,}000 = 10{,}000 / 500{,}000$ ✓

B ignores the leverage. C flips the sign of $(r_p - r_D)$. The portfolio earned less than the loan cost, so leverage hurt.

11 **B ✓ 50%**

payoff $= 129 - 120 = 9 \rightarrow (9 - 6) / 6 = \underline{50\%}$

A is the stock's own return (7.5%). C is payoff ÷ premium (150%): it forgets that the premium was paid.

12 **B ✓ Liquidity premium**

Selling quickly without moving the price is liquidity. A pays for the risk the issuer doesn't pay. C pays for longer bonds' sensitivity to interest rates.

13 **A ✓ The real risk-free rate**

Time preference alone: no default, no market risk. B pays for the chance the borrower doesn't pay; C for holding equities instead of the risk-free asset.

14 **A ✓ The dividend only**

The dividend was received this year. The 20% price gain is unrealized, and taxes usually follow realization.

SCORE YOURSELF

12–14: move on to Learning Module 2. 9–11: redo the worked examples behind your misses. Below 9: reread §2 and §3, which carry 10 of the module's 15 end-of-module questions.

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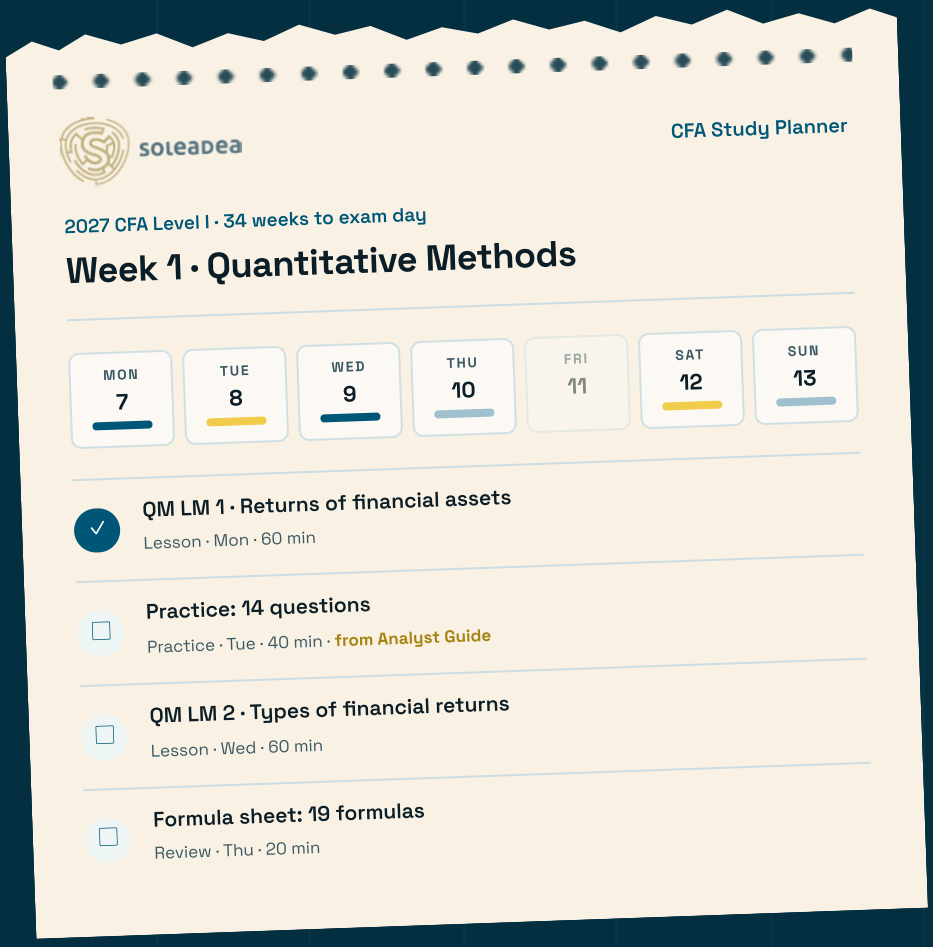


FIG. ∞ THE STUDY PLAN THESE NOTES BELONG TO.

Find the answer. Show your working.

analyst guide

Learning guide for the 2027 CFA® Program, Level I, Quantitative Methods, Learning Module 1: Returns of Financial Assets and Instruments.

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